Mixed numerical scheme solution for dropwise evaporative cooling

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ABSTRACT

A numerical code for the prediction of evaporative cooling of solid surfaces induced by a gently deposited water droplet is presented. The code is based upon a solid-liquid coupled model which predicts the droplet evaporation and the solid surface cooling for materials with thermal conductivity spanning over more than two orders of magnitude. The numerical solution of the conduction equation, which links a control volume method (CVM) used for the liquid and a boundary element method (BEM) used for the solid, is presented. The necessity of using the BEM for the solid domain is particularly stressed.

INTRODUCTION

The evaporation of a liquid droplet on a hot solid surface is a subject of practical interest in many industrial areas, such as spray cooling of solid in steel industries, vaporization process in internal combustion engines, cooling of turbine blades, and many others. The solid and liquid thermal behavior, the heat transfer phenomena involved and the relevant parameters governing the evaporative transient constitute the main objectives of the studies conducted in this field. Many experimental and theoretical investigations [1-5] have been carried out for liquid droplets on a high temperature solid surface. DiMarzo *et al.* [6,7] developed a mathematical model to describe the thermal transient due to a single droplet evaporating on a solid surface. In this model, an integral Control Volume Method (CVM) is applied for the droplet while Boundary Element Method (BEM) is used for the solid. Although the droplet and the solid are treated separately by different numerical methods, the temperature in the droplet and along the solid surface are solved

simultaneously at each time step by coupling the CVM and the BEM in the numerical model, which is presented in the following paragraphs.

GOVERNING EQUATION AND BOUNDARY CONDITIONS

The conduction equation can be written for both the liquid and the solid region, provided that the thermal diffusivity, α, is changed accordingly:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{1}$$

Three interfaces are involved in this evaporative cooling phenomenon: a) the liquid-air interface, b) the liquid-solid interface and c) the solid-air interface. The boundary conditions applying to each of them must be given to solve the governing equation. At the liquid-air interface one has:

$$-k_l \nabla T = h\,(T - T_a) + 0.624\, h_c \left(\frac{D}{\alpha_a}\right)^{\frac{2}{3}} \frac{\lambda_{fg}}{c_a} \frac{x_i - x_a}{1 - x_a} \tag{2}$$

At the liquid-solid interface, the continuity boundary conditions are:

$$\begin{aligned} T_l &= T_s \\ k_l \nabla T_l &= k_s \nabla T_s \end{aligned} \tag{3}$$

The boundary condition for the solid-air interface takes into account the convective and radiative heat transfer contributions:

$$k_s \nabla T = h(T_s - T_a) + \sigma \varepsilon (T_s^4 - T_a^4) \tag{4}$$

Because of the axisymmetric nature of the droplet, the gradient of temperature is zero on the vertical axis through the origin of either spherical or cylindrical coordinates. Furthermore, the gradient of temperature can be set equal to zero at points far away from the droplet since the evaporative cooling effect becomes negligible. When the droplet is deposited on the solid surface, the liquid and the solid have uniform temperature distribution. Therefore, from the physical point of view, uniform and constant temperature for the droplet and the solid surface respectively can be used as initial conditions.

NODALIZATION AND NUMERICAL METHODS

In order to approach numerically the thermal evaporative transient, an accurate discretization of the solid-liquid domain has to be chosen. Under the assumptions that the wetted area remains constant and that the droplet keeps a spherical segment shape during the whole transient, the volume of the droplet can be expressed as a function of its radius and

thickness. The appropriate coordinate system to be used is cylindrical. After some algebra, the coordinates (r,z) of all the points in the droplet can be defined for $0 \le r \le R$ and $0 \le z \le a$. In order to improve the accuracy of the model in the interfacial regions, two additional ("virtual") layers for the droplet are defined, one underneath the liquid-solid interface and the other above the liquid-air interface. The coordinates (r,z) of the points of the first virtual layer can be determined by symmetry about the interface with the liquid points immediately above it. The coordinates of the points of the second virtual layer are calculated as geometrically belonging to the droplet. The liquid droplet domain is discretized by a staggered grid, in which the geometric coordinates are defined at each corner of a mesh and the physical variable (temperature) is defined at the center (Fig. 1). A special nodalization is also required for the exposed solid surface. Since the temperature gradient near the droplet edge is very large, a refined discretization in this region is needed to describe the sharp variation of temperature. This refinement can be progressively reduced as the distance from the droplet edge becomes larger (Fig. 2). The cooling effect becomes negligible, for all materials, at a distance corresponding to 5-6 times the radius of the droplet; this result dictates the size of the domain of discretization in the code.

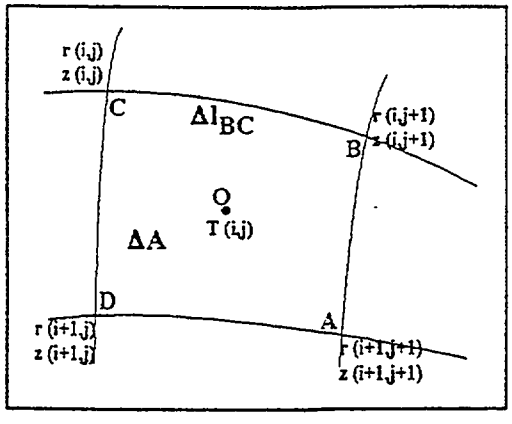

Figure 1

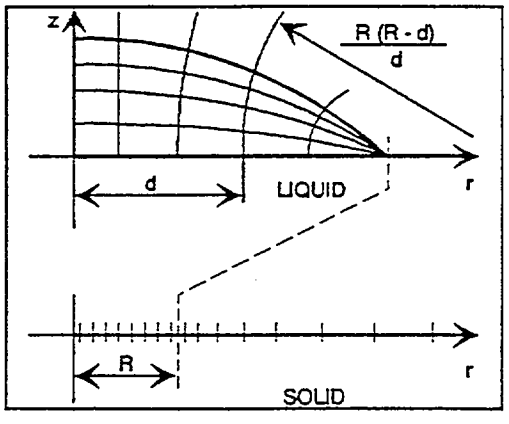

Figure 2

Two different numerical techniques are applied to study the droplet and the solid behaviors during the evaporative transient. CVM is used for the droplet, while the BEM is used for the solid. The CVM consists of an integration of the conduction equation over the liquid droplet. By integrating over the volume of the droplet and applying the Gauss' theorem for the right-hand-side, the discretized governing equation in cylindrical coordinates becomes:

$$r_0 \frac{\Delta T}{\Delta t} \Delta A = \alpha \sum_j \frac{\Delta T_j}{\Delta \bar{r}_j} r_j \Delta l_j \qquad (5)$$

where all the coefficients are time dependent since the droplet geometry

varies with time. By applying a Crank-Nicolson scheme, one can obtain an equation where all the T_{ij}'s at the time step $n+1$ are the only unknowns.

The BEM is used to solve the conduction equation in the solid (see also [8]). There are two advantages implicit in this methodology: a) since the relevant events take place on the surface of the domain, only this area is considered in the calculations; b) in order to analyze sharp localized changes in the temperature gradients, an integral approach is more effective than a method based on differentiation. To solve the conduction equation for the solid, an adjoint equation in terms of the Green's function G is defined as:

$$\frac{\partial G}{\partial t} = -\alpha \nabla^2 G \tag{6}$$

By multiplying Eqs. (1) and (6) by G and T respectively, then combining them together and integrating over the domain by using Gauss' theorem, one obtains:

$$\int_t \int_V \frac{\partial (TG)}{\partial t} dV\, dt = \alpha \int_t \oint_A (T\nabla G - G\nabla T)\cdot \vec{n}\, dA\, dt \tag{7}$$

The Green function is chosen as:

$$G(r,z,t,r_0,z_0,t_0) = (4\pi\alpha_s t_0)^{-\frac{3}{2}} \left(e^{-\frac{(r-r_0)^2+(z-z_0)^2}{4\pi t_0}} + e^{-\frac{(r-r_0)^2+(z+z_0)^2}{4\pi t_0}} \right) \tag{8}$$

and the following variables are also defined for convenience:

$$u = T - T_0 + \frac{q_0 z}{k_s} \quad ; \qquad f = \frac{du}{dz} \quad . \tag{9}$$

In cylindrical coordinates, the equation is further simplified since the angular integration can be expressed in terms of Bessel functions. The final forms results in a double integral in time and along the radius r:

$$u(r,t) = \frac{1}{\sqrt{4\pi\alpha}} \int_0^t \int_0^\infty \nabla u(r_0,t_0)\, r_0 t_0^{-\frac{3}{2}} L_0\left(\frac{2rr_0}{4\alpha t_0}\right) e^{-\frac{(r-r_0)^2}{4\alpha t_0}}\, dr_0 dt_0 \tag{10}$$

In order to simplify the task of handling these complex surface integrations, the following form is used:

$$u = \sum_{i=1}^{n} W_i \cdot \vec{f}_i + W_0 \cdot \vec{f}_0 \tag{11}$$

where W is a weight matrix and f is the vector of the forcing and unknown functions. The summation term is known since it involves previously calculated parameters. The second term on the right-hand-side contains unknown functions and the specified boundary conditions. Once the

weights are assigned for each pair of points they do not change throughout the computation. A new time scale must be introduced, which has its origin at the present time t and stretches its positive axis toward the past. Due to this set-up, t_0 is identified as the "recollection time". It represents the "memory" of the system in terms of heat fluxes. While the actual time is elapsing as the evaporative cooling process takes place, the recollection time is always zero at the present time. Then, since the effect of past forcing functions fades as actual time increases, hence the corresponding weights keep decreasing and after a few time steps they become negligible. This shows that the system recollection time can be considered for a limited number of time steps, even if its theoretical extension corresponds to the time elapsed from the beginning of the transient to the present time. From the computational point of view, this results in a summation in time which can be truncated as the corresponding values of the weight matrix become negligible. Since the weight function depends on the diffusivity of the material constituting the solid surface, the temporal sensibility of the system also varies according to the chosen materials. It is found that a recollection time of 10 to 20 seconds can be imposed in the code, from a conservative point of view, for any material. At the liquid-solid interface, the forcing function, f, constitutes the main parameter in the liquid-solid coupling, since it represents the term which accounts for the continuity of heat flux in terms of boundary conditions. By definition the forcing function can be written as $f = du/dz$, yielding two different expressions in the solid-air interfacial region and in the solid-liquid interfacial region.

In order to find the new coordinates for the droplet nodal points at each time step, the variation of the droplet volume has to be calculated by using the temperature distribution at the current time step, in particular the temperature along the liquid-air interface. After calculating the new droplet volume and coordinates, one can perform again all the computations concerning temperatures and heat fluxes until when the complete evaporation is reached. The variation of the droplet volume with time is obtained at each step by using the following expression:

$$\frac{dV}{dt} = \frac{2\pi(0.624)h}{\rho_w c_a}\left(\frac{D}{\alpha_a}\right)^{\frac{2}{3}} \int_0^L \frac{x_i - x_a}{1 - x_a} r\, dl \qquad (12)$$

and the new volume is calculated accordingly:

$$V_{new} = V_{old} - \frac{dV}{dt} \cdot \Delta t \qquad (13)$$

Once the new current volume of the droplet is given, another set of grid points for the droplet can be formed to proceed into the transient with the calculation of temperatures and fluxes. The cycle stops when the droplet volume becomes zero, i.e. when the droplet is completely evaporated.

CODE FORMULATION

The equations for T - written for the solid, the liquid and the interfacial regions - are regrouped to form a linear system that has to be solved at each time step during the transient. Although two different numerical methods are used for the droplet and the solid surface, they can be applied together to form the following matrix equation:

$$B \cdot \underline{T} = \underline{R} \tag{14}$$

where T is the vector of the temperatures at the current time step. The matrix B and the vector R contain the information on the evolution of the droplet shape and on the droplet temperature at the last time step in the liquid domain. On the solid surface, B becomes the matrix of the coefficients derived from the spatial and temporal integration required by the BEM, while R contains all the known surface temperatures and/or heat fluxes at the previous time steps which are taken into consideration depending on the chosen size of the recollection time. The dimension of the matrix B in the code is 216x216, resulting from the following choice of parameters: a 16x11 grid for the liquid, 40 points in the exposed solid surface. The matrix B is pentadiagonal in the liquid region and full in the solid region. In order to find a solution for T at each time step, the matrix of the coefficients, B, has to be inverted and multiplied by the vector of the known terms, R (a normalization of the matrix B is necessary in order to obtain a diagonal dominant coefficient matrix). After this intermediate step, the current temperature T can be calculated directly as:

$$\underline{T} = B^{-1} \cdot \underline{R} \tag{15}$$

From the mathematical point of view, there is a singularity when time is zero. Numerical fluctuations in temperature and heat flux can be expected if the initial temperature distribution is not properly chosen. In order to minimize these fluctuations, an iterative scheme can be used. The program runs for a small time interval (1 second) with very small time steps (no more than 0.1s each), until a reasonable temperature distribution in the droplet and in the solid is achieved; then, the calculations are restarted with this temperature distribution used as a new initial condition. This procedure is justified by the temporal characteristics of the evaporative transient, since the most relevant events occur in the final part of the process.

Two different constant values of the heat transfer coefficient are used in the code, one for the droplet and air interface, and another for the exposed solid surface. Both heat transfer coefficients are obtained from the investigation of experimental measurements. The assumption of constant heat transfer coefficients is reasonable for the regions where the temperature varies in a relatively small range along the surfaces, but it creates a mathematical discontinuity near the droplet edge. The

temperature in this region, both in the droplet and along the exposed solid surface, changes very rapidly. Some fluctuations in the temperature and in the heat flux can be expected because of the constant heat transfer coefficients applied near the edge.

CONCLUSIONS

A numerical code for the prediction of evaporative cooling of solid surfaces induced by a gently deposited water droplet has been presented. The code can be used to predict the evaporation time and the temperature distribution in the droplet and along the solid surface. Numerical tests have been conducted over a wide range of materials constituting the hot solid surface. Besides the numerical investigation, the model has been validated on the basis of the comparison between its numerical predictions and many experimental tests obtained with the infrared apparatus described in [9]. A direct comparison between numerical predictions and experimental results is possible for the evaporation time and for the distributions of temperature on the exposed solid surface. Therefore, a very large number of data from tests on aluminum and Macor has been collected, in a broad range of initial solid surface temperatures, and all the corresponding evaporation times have been calculated by the code and compared with the experimental values. The comparison between experimental data and code predictions has been extensively shown and discussed in [10]. A remarkable agreement has been observed, and, on the basis of the satisfactory results obtained by the single-droplet code, a multi-droplet model has been recently formulated.

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NOMENCLATURE

A	surface area
B	matrix of coefficients
c	specific heat
D	mass diffusivity
$\underline{f}$	forcing functions vector
G	Green's function
h	heat transfer coefficient
k	thermal conductivity
L_0	modified Bessel's function
l	length
q	heat flux
r	radial coordinate
R	wetted region radius
$\underline{R}$	vector of known terms
t	time
t_o	recollection time
T	temperature
u	transformed temperature
V	volume
W	weight matrix
x	steam mass fraction
z	axial coordinate

Greek letters

α	thermal diffusivity
ϵ	solid surface emissivity
λ_{fg}	latent heat of vaporization
ρ	density
σ	Stefan-Boltzmann constant
τ	evaporation time

subscripts

a	ambient, air
c	convective
i	interface
l	liquid
r	radial direction
s	solid
z	axial direction
0	initial

PART THREE

To be consistent with fire protection scenarios, the heat input should be by radiation from above the solid surface to simulate a fire environment. This introduces a dual evaporative mechanism: a) by direct radiant input at the liquid-vapor interface; and b) by conduction at the solid-liquid interface. Additionally, the heat input at the liquid-vapor interface has a very strong influence on the droplet shape since it changes the liquid surface tension by increasing its temperature at the liquid-vapor interface. These are the major results of this portion of the study:

1. The direct radiative input at the liquid-vapor interface relaxes the surface tension and the liquid spreads more on the solid surface. The droplet configuration exhibits a lower initial contact angle (i.e. at the droplet edge).
2. The flatter configuration of the water on the solid surface results in an early receding condition for the liquid. Therefore, the wetted region shrinks significantly during the later portion of the evaporative transient.
3. The heat input by direct radiation increases at a slower rate than the heat input by conduction. Therefore, conduction at the solid-liquid interface becomes the major contributor as the overall heat input increases.

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